Geometric Folding Algorithms: Linkages, Origami, Polyhedra [*DO07*]

Updates to Chapter 25, Section 25.1.2 Folding Polygons to Polyhedra

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Open Problem 25.1 (p. 384) asks:

> "Does every simple polygon fold (by perimeter gluing) to some simple polyhedron?"

The question was answered YES by Burago & Zalgaller's 1996 theorem [BZ96]:

Theorem 1 (Burago-Zalgaller) *"Every polyhedron M admits an isometric piecewise linear $\mathcal{C}^0$ immersion into $\mathbb{R}^3$ If M is orientable or has a nonempty boundary, then M admits an isometric piecewise linear $\mathcal{C}^0$ embedding into $\mathbb{R}^3$."*

Here is my "translation" (between two mathematical languages) of the embedding claim, as described in an unpublished 2010 note [O'R10]:

Theorem 2 *Every gluing of polygons to form an oriented manifold has an isometric embedding as a polyhedral surface (composed of a finite number of flat triangles) in $\mathbb{R}^3$.*

As Pak says [Pak10, p.334], this result can be viewed as a non-convex analogue of Alexandrov's existence theorem. We know of no upper bound on the number of triangles.

References

[BZ96] Yu. D. Burago and V. A. Zalgaller. Isometric piecewise linear immersions of two-dimensional manifolds with polyhedral metrics into $\mathbb{R}^3$. *St. Petersburg Math. J.*, 7(3):369–385, 1996. Translated by S. G. Ivanov.

[DO07] Erik D. Demaine and Joseph O'Rourke. *Geometric Folding Algorithms: Linkages, Origami, Polyhedra*. Cambridge University Press, 2007.

[O'R10] Joseph O'Rourke. On folding a polygon to a polyhedron. arXiv:1007.3181v1 [cs.CG]: arxiv.org/abs/1007.3181, July 2010.

[Pak10] Igor Pak. Lectures on Discrete and Polyhedral Geometry. http://www.math.ucla.edu/~pak/book.htm, 2010.