Geometric Folding Algorithms: Linkages, Origami, Polyhedra [DO07]

Updates to Chapter 23, Section 23.2: Flexible Polyhedra

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We report here on two updates. Recall from Section 23.2.3 that Steffen's 9-vertex flexible polyhedra was in several senses the best flexible polyhedron available, and, at least informally, believed likely optimal.

(1) Steffen's polyhedron has a dihedral angle flexing approximately 27°. A new flexible polyhedron flexes 29.2° [HG25]. Moreover, "all dihedral angles vary continuously during its motion." The polyhedron follows a "base + crinkle" construction

(2) Closing out a pursuit since Euler, and building on the work of many people—Connelly, Nelson, Maksimov—the authors prove that there is a flexible polyhedron on 8 vertices [GGLS]. And this is known to be the fewest number of vertices for any flexible polyhedron (Maksimov).

This breakthrough result has already been "improved" (has a wider range of mobility) [Atl25].

References

[Atl25] Elvar Atlason. Cutting along a symmetric quadrilateral to construct an embedded flexible dodecahedron. arXiv:2510.06897, 2025.

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[GGLS] Matteo Gallet, Georg Grasegger, Jan Legerskỳ, and Josef Schicho. Pentagonal bipyramids lead to the smallest flexible embedded polyhedron,(2024). arXiv:2410.13811.

[HG25] Zeyuan He and Simon D Guest. A new method for generalizing non-self-intersecting flexible polyhedra. arXiv:2505.05629, 2025.