Geometric Folding Algoritihms: Linkages, Origami, Polyhedra

Updates to Chapter 22, Section 22.4: Ununfoldable Polyhedra

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Abstract

Updates to Chapter 22, Edge-Unfolding of Polyhedra [DO07], specifically to Section 22.4, Ununfoldable Polyhedra.

1 Updates

Several new ununfoldable polyhedra have been discovered:

- Topologically convex ununfoldable polyhedra with 6 faces and 7 vertices, and with 7 faces and 6 vertices; see [ADE+22]. It's still open whether 5 or 6 faces/vertices are possible.
- "Very ununfoldable" polyhedra requiring many pieces, ununfoldable polyhedra with acute triangular faces, and ununfoldable polyhedra made by gluing together tetrahedra, are all in [DDE20]. This paper in particular provides a cleaner final argument for why cutting hats into multiple components must disconnect the polyhedron's surface, and can even force many pieces (see Theorems 3.1 and 5.1).

References

[ADE+22] Hugo A. Akitaya, Erik D. Demaine, David Eppstein, Tomohiro Tachi, and Ryuhei Uehara. Ununfoldable polyhedra with 66 vertices or 6 faces. *Computational Geometry*, 103:101857, 2022.

[DDE20] Erik D Demaine, Martin L Demaine, and David Eppstein. Acutely triangulated, stacked, and very ununfoldable polyhedra. In *32nd Canad. Conf. Comput. Geom.*, pages 106–113, 2020.

[DO07] Erik D. Demaine and Joseph O'Rourke. *Geometric Folding Algorithms: Linkages, Origami, Polyhedra*. Cambridge University Press, 2007.